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Introduction

1.1

THE ORIGINS OF OPERATIONS RESEARCH

Since the advent of the industrial revolution, the world has seen a remarkable growth in

the size and complexity of organizations. The artisans’ small shops of an earlier era have

evolved into the billion-dollar corporations of today. An integral part of this revolution-

ary change has been a tremendous increase in the division of labor and segmentation of

management responsibilities in these organizations. The results have been spectacular.

However, along with its blessings, this increasing specialization has created new prob-

lems, problems that are still occurring in many organizations. One problem is a tendency

for the many components of an organization to grow into relatively autonomous empires

with their own goals and value systems, thereby losing sight of how their activities and

objectives mesh with those of the overall organization. What is best for one component

frequently is detrimental to another, so the components may end up working at cross pur-

poses. A related problem is that as the complexity and specialization in an organization

increase, it becomes more and more difficult to allocate the available resources to the var-

ious activities in a way that is most effective for the organization as a whole. These kinds

of problems and the need to find a better way to solve them provided the environment for

the emergence of operations research (commonly referred to as OR).

The roots of OR can be traced back many decades, when early attempts were made

to use a scientific approach in the management of organizations. However, the beginning

of the activity called operations research has generally been attributed to the military ser-

vices early in World War II. Because of the war effort, there was an urgent need to allo-

cate scarce resources to the various military operations and to the activities within each

operation in an effective manner. Therefore, the British and then the U.S. military man-

agement called upon a large number of scientists to apply a scientific approach to deal-

ing with this and other strategic and tactical problems. In effect, they were asked to do

research on (military) operations. These teams of scientists were the first OR teams. By

developing effective methods of using the new tool of radar, these teams were instrumental

in winning the Air Battle of Britain. Through their research on how to better manage con-

voy and antisubmarine operations, they also played a major role in winning the Battle of

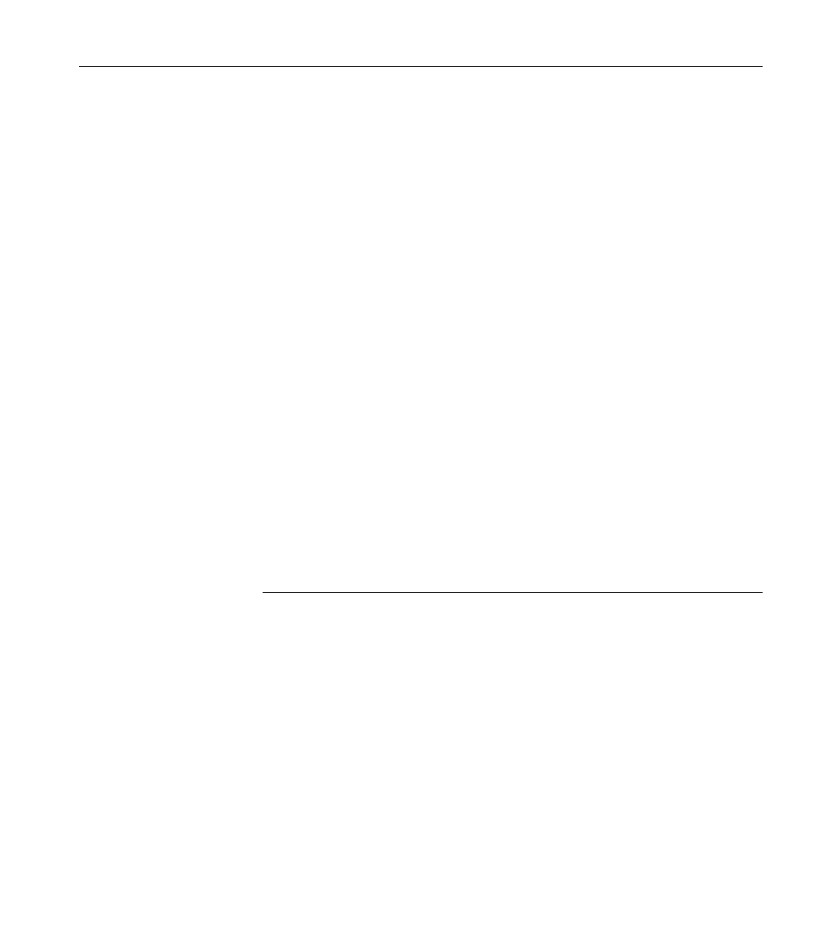
the North Atlantic. Similar efforts assisted the Island Campaign in the Pacific.

When the war ended, the success of OR in the war effort spurred interest in apply-

ing OR outside the military as well. As the industrial boom following the war was run-

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ning its course, the problems caused by the increasing complexity and specialization in

organizations were again coming to the forefront. It was becoming apparent to a growing

number of people, including business consultants who had served on or with the OR teams

during the war, that these were basically the same problems that had been faced by the

military but in a different context. By the early 1950s, these individuals had introduced

the use of OR to a variety of organizations in business, industry, and government. The

rapid spread of OR soon followed.

At least two other factors that played a key role in the rapid growth of OR during

this period can be identified. One was the substantial progress that was made early in im-

proving the techniques of OR. After the war, many of the scientists who had participated

on OR teams or who had heard about this work were motivated to pursue research rele-

vant to the field; important advancements in the state of the art resulted. A prime exam-

ple is the simplex method for solving linear programming problems, developed by George

Dantzig in 1947. Many of the standard tools of OR, such as linear programming, dynamic

programming, queueing theory, and inventory theory, were relatively well developed be-

fore the end of the 1950s.

A second factor that gave great impetus to the growth of the field was the onslaught

of the computer revolution. A large amount of computation is usually required to deal

most effectively with the complex problems typically considered by OR. Doing this by

hand would often be out of the question. Therefore, the development of electronic digital

computers, with their ability to perform arithmetic calculations thousands or even millions

of times faster than a human being can, was a tremendous boon to OR. A further boost

came in the 1980s with the development of increasingly powerful personal computers ac-

companied by good software packages for doing OR. This brought the use of OR within

the easy reach of much larger numbers of people. Today, literally millions of individuals

have ready access to OR software. Consequently, a whole range of computers from main-

frames to laptops now are being routinely used to solve OR problems.

1.2

THE NATURE OF OPERATIONS RESEARCH

As its name implies, operations research involves “research on operations.” Thus, opera-

tions research is applied to problems that concern how to conduct and coordinate the op-

erations (i.e., the activities) within an organization. The nature of the organization is es-

sentially immaterial, and, in fact, OR has been applied extensively in such diverse areas

as manufacturing, transportation, construction, telecommunications, financial planning,

health care, the military, and public services, to name just a few. Therefore, the breadth

of application is unusually wide.

The research part of the name means that operations research uses an approach that

resembles the way research is conducted in established scientific fields. To a considerable

extent, the scientific method is used to investigate the problem of concern. (In fact, the

term management science sometimes is used as a synonym for operations research.) In

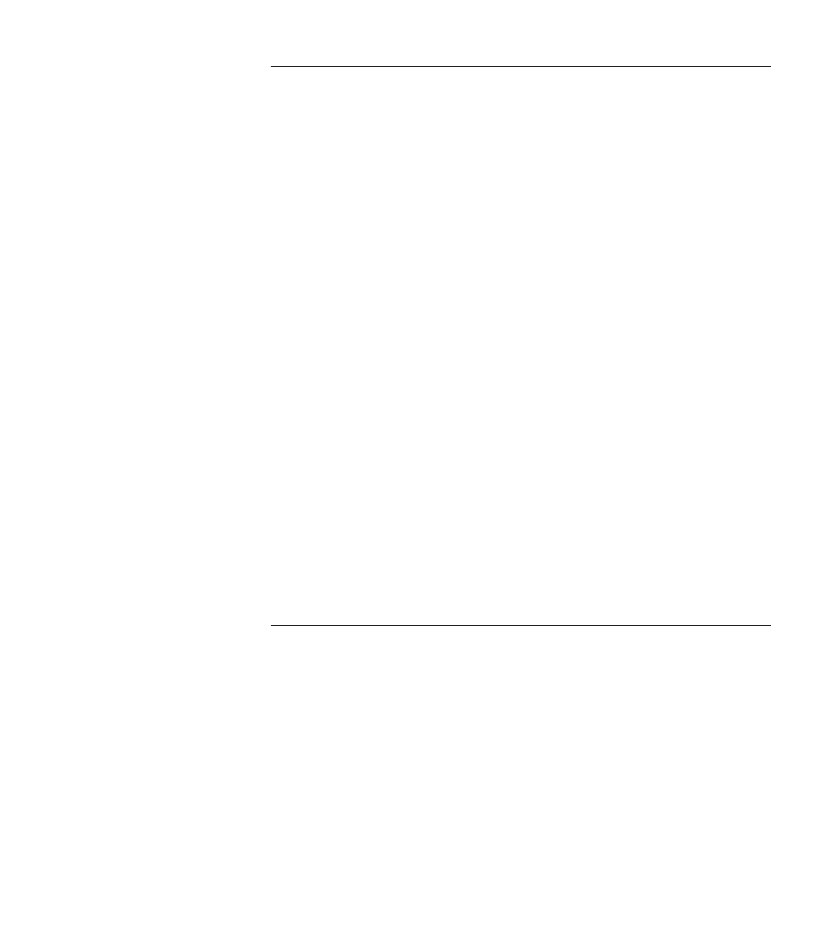
particular, the process begins by carefully observing and formulating the problem, in-

cluding gathering all relevant data. The next step is to construct a scientific (typically

mathematical) model that attempts to abstract the essence of the real problem. It is then

hypothesized that this model is a sufficiently precise representation of the essential fea-

tures of the situation that the conclusions (solutions) obtained from the model are also

1.3 THE IMPACT OF OPERATIONS RESEARCH

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valid for the real problem. Next, suitable experiments are conducted to test this hypothe-

sis, modify it as needed, and eventually verify some form of the hypothesis. (This step is

frequently referred to as model validation.) Thus, in a certain sense, operations research

involves creative scientific research into the fundamental properties of operations. How-

ever, there is more to it than this. Specifically, OR is also concerned with the practical

management of the organization. Therefore, to be successful, OR must also provide pos-

itive, understandable conclusions to the decision maker(s) when they are needed.

Still another characteristic of OR is its broad viewpoint. As implied in the preceding

section, OR adopts an organizational point of view. Thus, it attempts to resolve the con-

flicts of interest among the components of the organization in a way that is best for the

organization as a whole. This does not imply that the study of each problem must give

explicit consideration to all aspects of the organization; rather, the objectives being sought

must be consistent with those of the overall organization.

An additional characteristic is that OR frequently attempts to find a best solution (re-

ferred to as an optimal solution) for the problem under consideration. (We say a best in-

stead of the best solution because there may be multiple solutions tied as best.) Rather

than simply improving the status quo, the goal is to identify a best possible course of ac-

tion. Although it must be interpreted carefully in terms of the practical needs of manage-

ment, this “search for optimality” is an important theme in OR.

All these characteristics lead quite naturally to still another one. It is evident that no

single individual should be expected to be an expert on all the many aspects of OR work

or the problems typically considered; this would require a group of individuals having di-

verse backgrounds and skills. Therefore, when a full-fledged OR study of a new problem

is undertaken, it is usually necessary to use a team approach. Such an OR team typically

needs to include individuals who collectively are highly trained in mathematics, statistics

and probability theory, economics, business administration, computer science, engineering

and the physical sciences, the behavioral sciences, and the special techniques of OR. The

team also needs to have the necessary experience and variety of skills to give appropriate

consideration to the many ramifications of the problem throughout the organization.

1.3

THE IMPACT OF OPERATIONS RESEARCH

Operations research has had an impressive impact on improving the efficiency of numer-

ous organizations around the world. In the process, OR has made a significant contribu-

tion to increasing the productivity of the economies of various countries. There now are

a few dozen member countries in the International Federation of Operational Research

Societies (IFORS), with each country having a national OR society. Both Europe and Asia

have federations of OR societies to coordinate holding international conferences and pub-

lishing international journals in those continents.

It appears that the impact of OR will continue to grow. For example, according to the

U.S. Bureau of Labor Statistics, OR currently is one of the fastest-growing career areas

for U.S. college graduates.

To give you a better notion of the wide applicability of OR, we list some actual award-

winning applications in Table 1.1. Note the diversity of organizations and applications in

the first two columns. The curious reader can find a complete article describing each ap-

plication in the January–February issue of Interfaces for the year cited in the third col-

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TABLE 1.1 Some applications of operations research

Organization

The Netherlands

Rijkswaterstaat

Monsanto Corp.

Nature of Application

Develop national water management

policy, including mix of new facilities,

operating procedures, and pricing.

Optimize production operations in

chemical plants to meet production targets

with minimum cost.

Schedule shift work at reservation offices

and airports to meet customer needs with

minimum cost.

Optimize refinery operations and the supply,

distribution, and marketing of products.

Optimally schedule and deploy police

patrol officers with a computerized system.

Optimally blend available ingredients into

gasoline products to meet quality and

sales requirements.

Integrate a national network of spare parts

inventories to improve service support.

Optimize the design of a national trucking

network and the routing of shipments.

Design an effective needle exchange

program to combat the spread of HIV/AIDS.

Develop a PC-based system to guide

business customers in designing their call

centers.

Maximize the profit from assigning

airplane types to over 2500 domestic

flights.

Restructure the global supply chain of

suppliers, plants, distribution centers,

potential sites, and market areas.

Optimally select and schedule massive

projects for meeting the country’s future

energy needs.

Optimally redesign the size and shape of

the defense force and its weapons systems.

Redesign the North American production

and distribution system to reduce costs

and improve speed to market.

Optimally schedule employees to provide

desired customer service at a minimum

cost.

Redesign the sizes and locations of

buffers in a printer production line to meet

production goals.

Year of

Publication\*

1985

Related

Chapters†

2–8, 13, 22

Annual

Savings

$15 million

1985

2, 12

$2 million

United Airlines

1986

2–9, 12, 17,

18, 20

2–9, 20

2–4, 12, 20

2, 13

$6 million

Citgo Petroleum

Corp.

San Francisco

Police Department

Texaco, Inc.

1987

1989

1989

$70 million

$11 million

$30 million

IBM

1990

2, 19, 22

Yellow Freight

System, Inc.

New Haven Health

Department

AT&T

1992

1993

1993

2, 9, 13, 20,

22

2

17, 18, 22

$20 million

$250 million

less inventory

$17.3 million

33% less

HIV/AIDS

$750 million

Delta Airlines

1994

12

$100 million

Digital Equipment

Corp.

China

1995

12

$800 million

1995

12

$425 million

South African

defense force

Proctor and Gamble

1997

1997

12

8

$1.1 billion

$200 million

Taco Bell

1998

12, 20, 22

$13 million

Hewlett-Packard

1998

17, 18

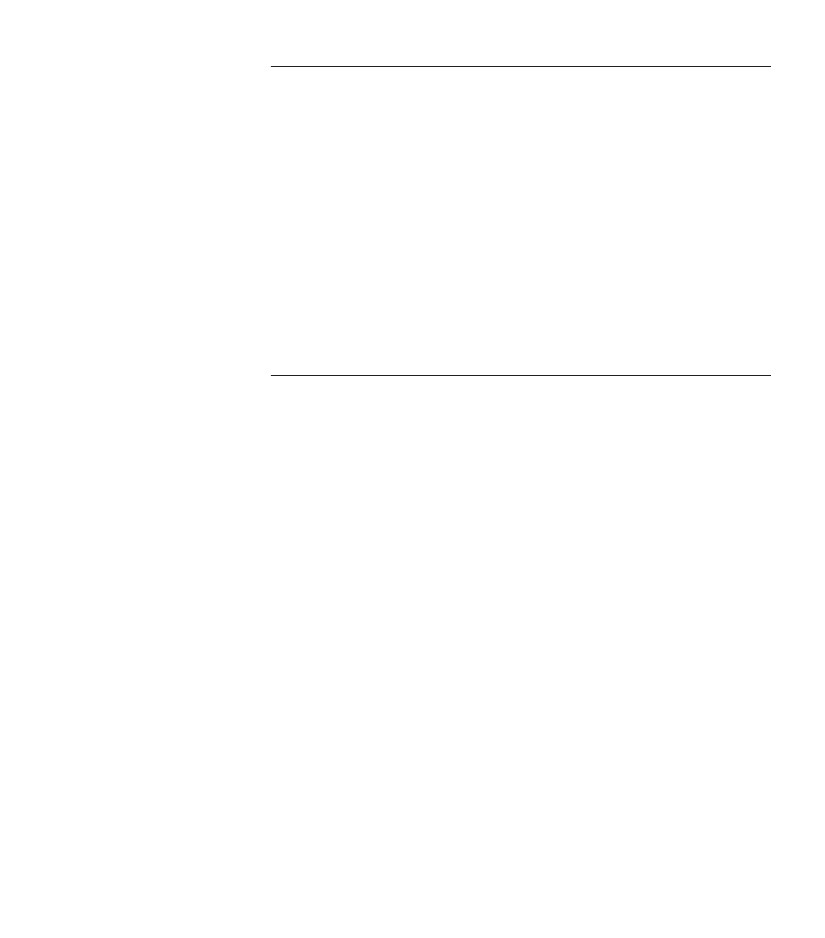
$280 million

more revenue

\*Pertains to a January–February issue of Interfaces in which a complete article can be found describing the application.

†Refers to chapters in this book that describe the kinds of OR techniques used in the application.

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1.4 ALGORITHMS AND OR COURSEWARE

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umn of the table. The fourth column lists the chapters in this book that describe the kinds

of OR techniques that were used in the application. (Note that many of the applications

combine a variety of techniques.) The last column indicates that these applications typi-

cally resulted in annual savings in the millions (or even tens of millions) of dollars. Fur-

thermore, additional benefits not recorded in the table (e.g., improved service to customers

and better managerial control) sometimes were considered to be even more important than

these financial benefits. (You will have an opportunity to investigate these less tangible

benefits further in Probs. 1.3-1 and 1.3-2.)

Although most routine OR studies provide considerably more modest benefits than

these award-winning applications, the figures in the rightmost column of Table 1.1 do ac-

curately reflect the dramatic impact that large, well-designed OR studies occasionally can

have.

We will briefly describe some of these applications in the next chapter, and then we

present two in greater detail as case studies in Sec. 3.5.

1.4

ALGORITHMS AND OR COURSEWARE

An important part of this book is the presentation of the major algorithms (systematic

solution procedures) of OR for solving certain types of problems. Some of these algo-

rithms are amazingly efficient and are routinely used on problems involving hundreds or

thousands of variables. You will be introduced to how these algorithms work and what

makes them so efficient. You then will use these algorithms to solve a variety of problems

on a computer. The CD-ROM called OR Courseware that accompanies the book will be

a key tool for doing all this.

One special feature in your OR Courseware is a program called OR Tutor. This pro-

gram is intended to be your personal tutor to help you learn the algorithms. It consists of

many demonstration examples that display and explain the algorithms in action. These

“demos” supplement the examples in the book.

In addition, your OR Courseware includes many interactive routines for executing

the algorithms interactively in a convenient spreadsheet format. The computer does all the

routine calculations while you focus on learning and executing the logic of the algorithm.

You should find these interactive routines a very efficient and enlightening way of doing

many of your homework problems.

In practice, the algorithms normally are executed by commercial software packages.

We feel that it is important to acquaint students with the nature of these packages that

they will be using after graduation. Therefore, your OR Courseware includes a wealth of

material to introduce you to three particularly popular software packages described be-

low. Together, these packages will enable you to solve nearly all the OR models encoun-

tered in this book very efficiently. We have added our own automatic routines to the OR

Courseware only in a few cases where these packages are not applicable.

A very popular approach now is to use today’s premier spreadsheet package, Mi-

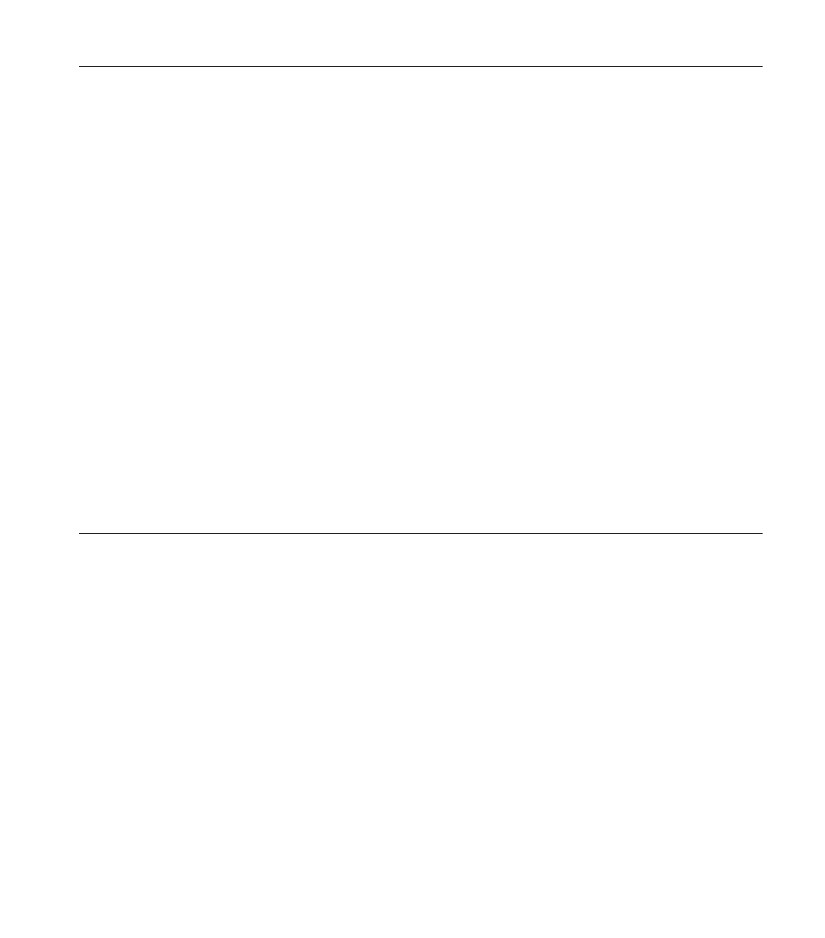
crosoft Excel, to formulate small OR models in a spreadsheet format. The Excel Solver

then is used to solve the models. Your OR Courseware includes a separate Excel file for

nearly every chapter in this book. Each time a chapter presents an example that can be

solved using Excel, the complete spreadsheet formulation and solution is given in that

chapter’s Excel file. For many of the models in the book, an Excel template also is pro-

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vided that already includes all the equations necessary to solve the model. Some Excel

add-ins also are included on the CD-ROM.

After many years, LINDO (and its companion modeling language LINGO) contin-

ues to be a dominant OR software package. Student versions of LINDO and LINGO now

can be downloaded free from the Web. As for Excel, each time an example can be solved

with this package, all the details are given in a LINGO/LINDO file for that chapter in

your OR Courseware.

CPLEX is an elite state-of-the-art software package that is widely used for solving

large and challenging OR problems. When dealing with such problems, it is common to

also use a modeling system to efficiently formulate the mathematical model and enter it

into the computer. MPL is a user-friendly modeling system that uses CPLEX as its main

solver. A student version of MPL and CPLEX is available free by downloading it from

the Web. For your convenience, we also have included this student version in your OR

Courseware. Once again, all the examples that can be solved with this package are de-

tailed in MPL/CPLEX files for the corresponding chapters in your OR Courseware.

We will further describe these three software packages and how to use them later (es-

pecially near the end of Chaps. 3 and 4). Appendix 1 also provides documentation for the

OR Courseware, including OR Tutor.

To alert you to relevant material in OR Courseware, the end of each chapter from

Chap. 3 onward has a list entitled Learning Aids for This Chapter in Your OR Course-

ware. As explained at the beginning of the problem section for each of these chapters,

symbols also are placed to the left of each problem number or part where any of this ma-

terial (including demonstration examples and interactive routines) can be helpful.

PROBLEMS

1.3-1. Select one of the applications of operations research listed

in Table 1.1. Read the article describing the application in the

January–February issue of Interfaces for the year indicated in the

third column. Write a two-page summary of the application and

the benefits (including nonfinancial benefits) it provided.

1.3-2. Select three of the applications of operations research listed

in Table 1.1. Read the articles describing the applications in the Jan-

uary–February issue of Interfaces for the years indicated in the third

column. For each one, write a one-page summary of the applica-

tion and the benefits (including nonfinancial benefits) it provided.

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Overview of the

Operations Research

Modeling Approach

The bulk of this book is devoted to the mathematical methods of operations research (OR).

This is quite appropriate because these quantitative techniques form the main part of what

is known about OR. However, it does not imply that practical OR studies are primarily

mathematical exercises. As a matter of fact, the mathematical analysis often represents only

a relatively small part of the total effort required. The purpose of this chapter is to place

things into better perspective by describing all the major phases of a typical OR study.

One way of summarizing the usual (overlapping) phases of an OR study is the

following:

1. Define the problem of interest and gather relevant data.

2. Formulate a mathematical model to represent the problem.

3. Develop a computer-based procedure for deriving solutions to the problem from the

model.

4. Test the model and refine it as needed.

5. Prepare for the ongoing application of the model as prescribed by management.

6. Implement.

Each of these phases will be discussed in turn in the following sections.

Most of the award-winning OR studies introduced in Table 1.1 provide excellent ex-

amples of how to execute these phases well. We will intersperse snippets from these ex-

amples throughout the chapter, with references to invite your further reading.

2.1

DEFINING THE PROBLEM AND GATHERING DATA

In contrast to textbook examples, most practical problems encountered by OR teams are

initially described to them in a vague, imprecise way. Therefore, the first order of busi-

ness is to study the relevant system and develop a well-defined statement of the problem

to be considered. This includes determining such things as the appropriate objectives, con-

straints on what can be done, interrelationships between the area to be studied and other

areas of the organization, possible alternative courses of action, time limits for making a

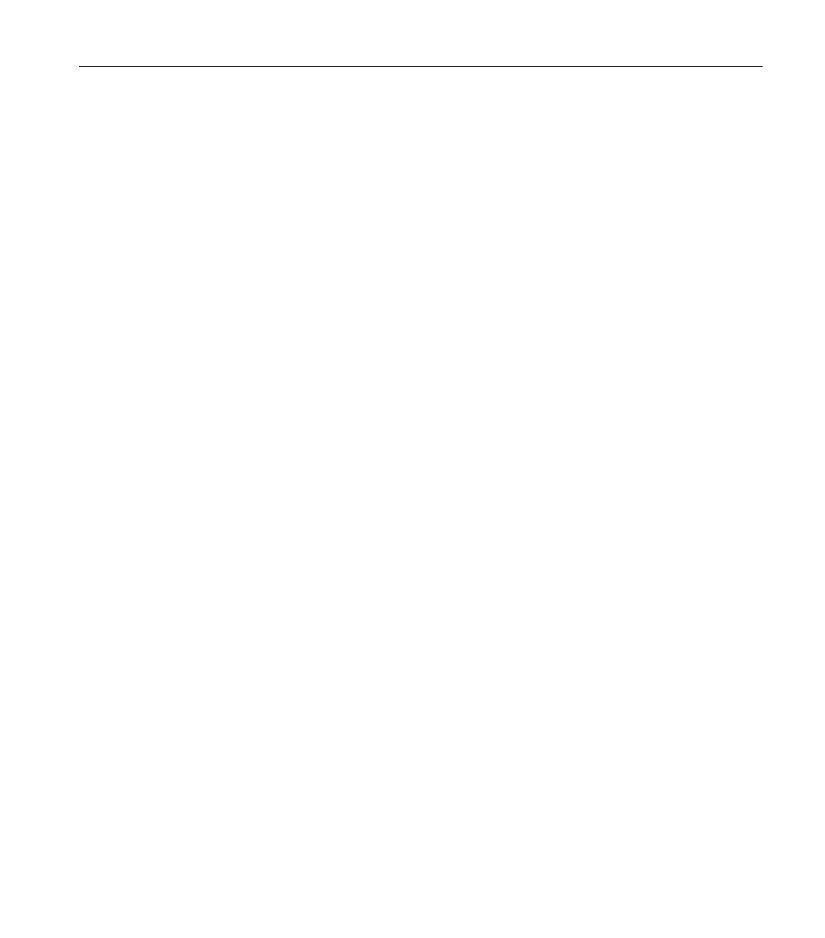
decision, and so on. This process of problem definition is a crucial one because it greatly

affects how relevant the conclusions of the study will be. It is difficult to extract a “right”

answer from the “wrong” problem!

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The first thing to recognize is that an OR team is normally working in an advisory ca-

pacity. The team members are not just given a problem and told to solve it however they

see fit. Instead, they are advising management (often one key decision maker). The team

performs a detailed technical analysis of the problem and then presents recommendations

to management. Frequently, the report to management will identify a number of alterna-

tives that are particularly attractive under different assumptions or over a different range of

values of some policy parameter that can be evaluated only by management (e.g., the trade-

off between cost and benefits). Management evaluates the study and its recommendations,

takes into account a variety of intangible factors, and makes the final decision based on its

best judgment. Consequently, it is vital for the OR team to get on the same wavelength as

management, including identifying the “right” problem from management’s viewpoint, and

to build the support of management for the course that the study is taking.

Ascertaining the appropriate objectives is a very important aspect of problem defini-

tion. To do this, it is necessary first to identify the member (or members) of management

who actually will be making the decisions concerning the system under study and then to

probe into this individual’s thinking regarding the pertinent objectives. (Involving the de-

cision maker from the outset also is essential to build her or his support for the imple-

mentation of the study.)

By its nature, OR is concerned with the welfare of the entire organization rather than

that of only certain of its components. An OR study seeks solutions that are optimal for

the overall organization rather than suboptimal solutions that are best for only one com-

ponent. Therefore, the objectives that are formulated ideally should be those of the entire

organization. However, this is not always convenient. Many problems primarily concern

only a portion of the organization, so the analysis would become unwieldy if the stated ob-

jectives were too general and if explicit consideration were given to all side effects on the

rest of the organization. Instead, the objectives used in the study should be as specific as

they can be while still encompassing the main goals of the decision maker and maintain-

ing a reasonable degree of consistency with the higher-level objectives of the organization.

For profit-making organizations, one possible approach to circumventing the prob-

lem of suboptimization is to use long-run profit maximization (considering the time value

of money) as the sole objective. The adjective long-run indicates that this objective pro-

vides the flexibility to consider activities that do not translate into profits immediately

(e.g., research and development projects) but need to do so eventually in order to be worth-

while. This approach has considerable merit. This objective is specific enough to be used

conveniently, and yet it seems to be broad enough to encompass the basic goal of profit-

making organizations. In fact, some people believe that all other legitimate objectives can

be translated into this one.

However, in actual practice, many profit-making organizations do not use this ap-

proach. A number of studies of U.S. corporations have found that management tends to

adopt the goal of satisfactory profits, combined with other objectives, instead of focusing

on long-run profit maximization. Typically, some of these other objectives might be to

maintain stable profits, increase (or maintain) one’s share of the market, provide for prod-

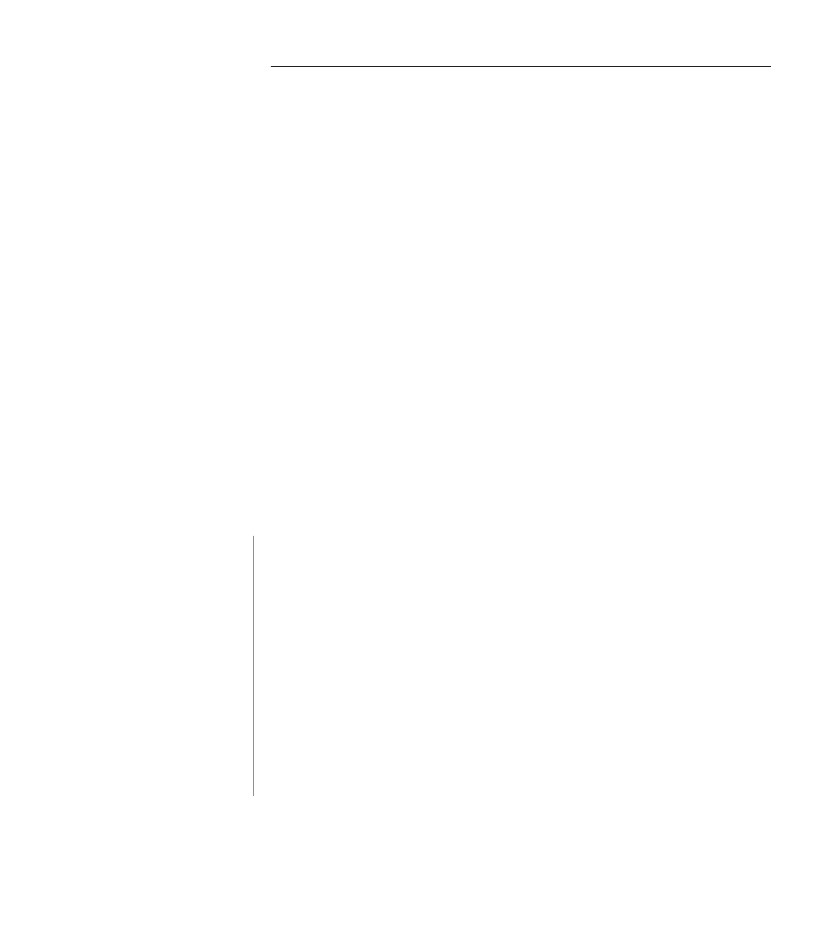
uct diversification, maintain stable prices, improve worker morale, maintain family con-

trol of the business, and increase company prestige. Fulfilling these objectives might

achieve long-run profit maximization, but the relationship may be sufficiently obscure that

it may not be convenient to incorporate them all into this one objective.

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2.1 DEFINING THE PROBLEM AND GATHERING DATA

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Furthermore, there are additional considerations involving social responsibilities that

are distinct from the profit motive. The five parties generally affected by a business firm

located in a single country are (1) the owners (stockholders, etc.), who desire profits (div-

idends, stock appreciation, and so on); (2) the employees, who desire steady employment

at reasonable wages; (3) the customers, who desire a reliable product at a reasonable price;

(4) the suppliers, who desire integrity and a reasonable selling price for their goods; and

(5) the government and hence the nation, which desire payment of fair taxes and consid-

eration of the national interest. All five parties make essential contributions to the firm,

and the firm should not be viewed as the exclusive servant of any one party for the ex-

ploitation of others. By the same token, international corporations acquire additional obli-

gations to follow socially responsible practices. Therefore, while granting that manage-

ment’s prime responsibility is to make profits (which ultimately benefits all five parties),

we note that its broader social responsibilities also must be recognized.

OR teams typically spend a surprisingly large amount of time gathering relevant data

about the problem. Much data usually are needed both to gain an accurate understanding

of the problem and to provide the needed input for the mathematical model being formu-

lated in the next phase of study. Frequently, much of the needed data will not be available

when the study begins, either because the information never has been kept or because what

was kept is outdated or in the wrong form. Therefore, it often is necessary to install a new

computer-based management information system to collect the necessary data on an on-

going basis and in the needed form. The OR team normally needs to enlist the assistance

of various other key individuals in the organization to track down all the vital data. Even

with this effort, much of the data may be quite “soft,” i.e., rough estimates based only on

educated guesses. Typically, an OR team will spend considerable time trying to improve

the precision of the data and then will make do with the best that can be obtained.

Examples. An OR study done for the San Francisco Police Department1 resulted in

the development of a computerized system for optimally scheduling and deploying police

patrol officers. The new system provided annual savings of $11 million, an annual $3 mil-

lion increase in traffic citation revenues, and a 20 percent improvement in response times.

In assessing the appropriate objectives for this study, three fundamental objectives were

identified:

1. Maintain a high level of citizen safety.

2. Maintain a high level of officer morale.

3. Minimize the cost of operations.

To satisfy the first objective, the police department and city government jointly established

a desired level of protection. The mathematical model then imposed the requirement that

this level of protection be achieved. Similarly, the model imposed the requirement of bal-

ancing the workload equitably among officers in order to work toward the second objec-

tive. Finally, the third objective was incorporated by adopting the long-term goal of min-

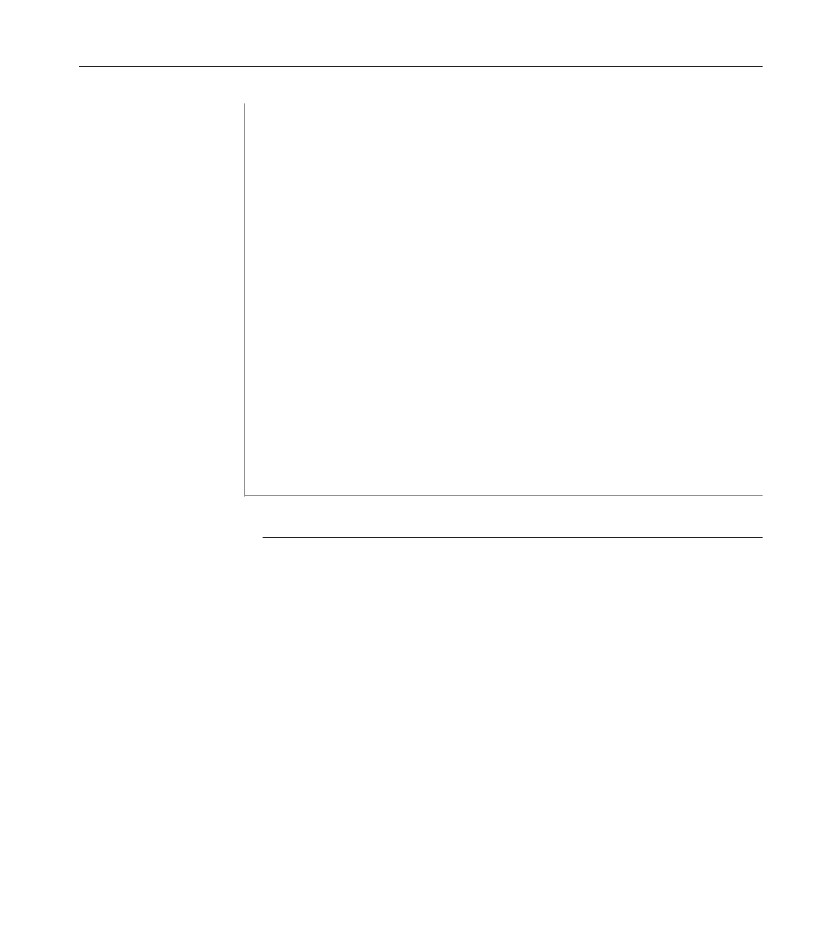
imizing the number of officers needed to meet the first two objectives.

1

P. E. Taylor and S. J. Huxley, “A Break from Tradition for the San Francisco Police: Patrol Officer Schedul-

ing Using an Optimization-Based Decision Support System,” Interfaces, 19(1): 4–24, Jan.–Feb. 1989. See es-

pecially pp. 4–11.

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The Health Department of New Haven, Connecticut used an OR team1 to de-

sign an effective needle exchange program to combat the spread of the virus that causes

AIDS (HIV), and succeeded in reducing the HIV infection rate among program clients

by 33 percent. The key part of this study was an innovative data collection program

to obtain the needed input for mathematical models of HIV transmission. This program

involved complete tracking of each needle (and syringe), including the identity, loca-

tion, and date for each person receiving the needle and each person returning the

needle during an exchange, as well as testing whether the returned needle was HIV-

positive or HIV-negative.

An OR study done for the Citgo Petroleum Corporation2 optimized both refinery

operations and the supply, distribution, and marketing of its products, thereby achieving

a profit improvement of approximately $70 million per year. Data collection also played

a key role in this study. The OR team held data requirement meetings with top Citgo man-

agement to ensure the eventual and continual quality of data. A state-of-the-art manage-

ment database system was developed and installed on a mainframe computer. In cases

where needed data did not exist, LOTUS 1-2-3 screens were created to help operations

personnel input the data, and then the data from the personal computers (PCs) were up-

loaded to the mainframe computer. Before data was inputted to the mathematical model,

a preloader program was used to check for data errors and inconsistencies. Initially, the

preloader generated a paper log of error messages 1 inch thick! Eventually, the number

of error and warning messages (indicating bad or questionable numbers) was reduced to

less than 10 for each new run.

We will describe the overall Citgo study in much more detail in Sec. 3.5.

2.2

FORMULATING A MATHEMATICAL MODEL

After the decision maker’s problem is defined, the next phase is to reformulate this prob-

lem in a form that is convenient for analysis. The conventional OR approach for doing

this is to construct a mathematical model that represents the essence of the problem. Be-

fore discussing how to formulate such a model, we first explore the nature of models in

general and of mathematical models in particular.

Models, or idealized representations, are an integral part of everyday life. Common

examples include model airplanes, portraits, globes, and so on. Similarly, models play an

important role in science and business, as illustrated by models of the atom, models of

genetic structure, mathematical equations describing physical laws of motion or chemical

reactions, graphs, organizational charts, and industrial accounting systems. Such models

are invaluable for abstracting the essence of the subject of inquiry, showing interrelation-

ships, and facilitating analysis.

E. H. Kaplan and E. O’Keefe, “Let the Needles Do the Talking! Evaluating the New Haven Needle Exchange,”

Interfaces, 23(1): 7–26, Jan.–Feb. 1993. See especially pp. 12–14.

2D. Klingman, N. Phillips, D. Steiger, R. Wirth, and W. Young, “The Challenges and Success Factors in Im-

plementing an Integrated Products Planning System for Citgo,” Interfaces, 16(3): 1–19, May–June 1986. See

especially pp. 11–14. Also see D. Klingman, N. Phillips, D. Steiger, and W. Young, “The Successful Deploy-

ment of Management Science throughout Citgo Petroleum Corporation,” Interfaces, 17(1): 4–25, Jan.–Feb. 1987.

See especially pp. 13–15. This application will be described further in Sec. 3.5.

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2.2 FORMULATING A MATHEMATICAL MODEL

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Mathematical models are also idealized representations, but they are expressed in

terms of mathematical symbols and expressions. Such laws of physics as F ma and

E mc2 are familiar examples. Similarly, the mathematical model of a business problem

is the system of equations and related mathematical expressions that describe the essence

of the problem. Thus, if there are n related quantifiable decisions to be made, they are

represented as decision variables (say, x1, x2, . . . , xn) whose respective values are to be

determined. The appropriate measure of performance (e.g., profit) is then expressed as a

mathematical function of these decision variables (for example, P 3x1 2x2 +5xn).

This function is called the objective function. Any restrictions on the values that can be

assigned to these decision variables are also expressed mathematically, typically by means

of inequalities or equations (for example, x1 3x1x2 2x2 10). Such mathematical ex-

pressions for the restrictions often are called constraints. The constants (namely, the co-

efficients and right-hand sides) in the constraints and the objective function are called the

parameters of the model. The mathematical model might then say that the problem is to

choose the values of the decision variables so as to maximize the objective function, sub-

ject to the specified constraints. Such a model, and minor variations of it, typifies the mod-

els used in OR.

Determining the appropriate values to assign to the parameters of the model (one

value per parameter) is both a critical and a challenging part of the model-building process.

In contrast to textbook problems where the numbers are given to you, determining param-

eter values for real problems requires gathering relevant data. As discussed in the pre-

ceding section, gathering accurate data frequently is difficult. Therefore, the value assigned

to a parameter often is, of necessity, only a rough estimate. Because of the uncertainty

about the true value of the parameter, it is important to analyze how the solution derived

from the model would change (if at all) if the value assigned to the parameter were changed

to other plausible values. This process is referred to as sensitivity analysis, as discussed

further in the next section (and much of Chap. 6).

Although we refer to “the” mathematical model of a business problem, real problems

normally don’t have just a single “right” model. Section 2.4 will describe how the process

of testing a model typically leads to a succession of models that provide better and bet-

ter representations of the problem. It is even possible that two or more completely dif-

ferent types of models may be developed to help analyze the same problem.

You will see numerous examples of mathematical models throughout the remainder

of this book. One particularly important type that is studied in the next several chapters

is the linear programming model, where the mathematical functions appearing in both

the objective function and the constraints are all linear functions. In the next chapter, spe-

cific linear programming models are constructed to fit such diverse problems as deter-

mining (1) the mix of products that maximizes profit, (2) the design of radiation therapy

that effectively attacks a tumor while minimizing the damage to nearby healthy tissue,

(3) the allocation of acreage to crops that maximizes total net return, and (4) the combi-

nation of pollution abatement methods that achieves air quality standards at minimum cost.

Mathematical models have many advantages over a verbal description of the problem.

One advantage is that a mathematical model describes a problem much more concisely. This

tends to make the overall structure of the problem more comprehensible, and it helps to re-

veal important cause-and-effect relationships. In this way, it indicates more clearly what ad-

ditional data are relevant to the analysis. It also facilitates dealing with the problem in its

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entirety and considering all its interrelationships simultaneously. Finally, a mathematical

model forms a bridge to the use of high-powered mathematical techniques and computers

to analyze the problem. Indeed, packaged software for both personal computers and main-

frame computers has become widely available for solving many mathematical models.

However, there are pitfalls to be avoided when you use mathematical models. Such a

model is necessarily an abstract idealization of the problem, so approximations and sim-

plifying assumptions generally are required if the model is to be tractable (capable of be-

ing solved). Therefore, care must be taken to ensure that the model remains a valid repre-

sentation of the problem. The proper criterion for judging the validity of a model is whether

the model predicts the relative effects of the alternative courses of action with sufficient

accuracy to permit a sound decision. Consequently, it is not necessary to include unim-

portant details or factors that have approximately the same effect for all the alternative

courses of action considered. It is not even necessary that the absolute magnitude of the

measure of performance be approximately correct for the various alternatives, provided that

their relative values (i.e., the differences between their values) are sufficiently precise. Thus,

all that is required is that there be a high correlation between the prediction by the model

and what would actually happen in the real world. To ascertain whether this requirement

is satisfied, it is important to do considerable testing and consequent modifying of the

model, which will be the subject of Sec. 2.4. Although this testing phase is placed later in

the chapter, much of this model validation work actually is conducted during the model-

building phase of the study to help guide the construction of the mathematical model.

In developing the model, a good approach is to begin with a very simple version and

then move in evolutionary fashion toward more elaborate models that more nearly reflect

the complexity of the real problem. This process of model enrichment continues only as

long as the model remains tractable. The basic trade-off under constant consideration is

between the precision and the tractability of the model. (See Selected Reference 6 for a

detailed description of this process.)

A crucial step in formulating an OR model is the construction of the objective function.

This requires developing a quantitative measure of performance relative to each of the deci-

sion maker’s ultimate objectives that were identified while the problem was being defined.

If there are multiple objectives, their respective measures commonly are then transformed

and combined into a composite measure, called the overall measure of performance. This

overall measure might be something tangible (e.g., profit) corresponding to a higher goal of

the organization, or it might be abstract (e.g., utility). In the latter case, the task of develop-

ing this measure tends to be a complex one requiring a careful comparison of the objectives

and their relative importance. After the overall measure of performance is developed, the ob-

jective function is then obtained by expressing this measure as a mathematical function of

the decision variables. Alternatively, there also are methods for explicitly considering multi-

ple objectives simultaneously, and one of these (goal programming) is discussed in Chap. 7.

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